PHYSICAL REVIEW E VOLUME 60, NUMBER 6 DECEMBER 1999

Liquid flow through aqueous foams: From the plateau border-dominated regime to the node-dominated regime

M. Durand, G. Martinoty, and D. Langevin
Laboratoire de Physique des Solides, Bâtiment 510, Université Paris-Sud, 91405 Orsay Cedex, France

(Received 20 August 1999)

The velocity of gravity-driven flow through aqueous foams (forced drainage) has been determined by using electrical conductivity measurements in foams made with solutions of different surfactants. There is always a scaling behavior (power law) between the drainage velocity V and the imposed flow rate Q: $V \sim Q^{\alpha}$, but the α coefficient varies between the different surfactant solutions and increases with surface viscosity. An explanation of this behavior will be given in terms of a transition between a node-dominated and a Plateau border-dominated viscous dissipation, for which theory predicts respectively $\alpha = \frac{1}{3}$ and $\alpha = \frac{1}{2}$.

[S1063-651X(99)51612-7]

PACS number(s): 47.55.Mh, 82.70.Rr, 68.10.Et

Drainage of aqueous foams is an old problem, still far from being completely understood. Recent studies focused on gravity-driven flow through aqueous foams with continuous supply of liquid, a process called ''forced drainage'' [1–4]. There is a difference with flow through porous media, since the liquid channels between bubbles (Plateau borders) expand when liquid is forced into the foam. Earlier theories assumed rigid walls and Poiseuille flow throughout the network of Plateau borders. The velocity of the liquid front V is then found to be constant (soliton wave) and proportional to Q^{α}, where Q is the flow rate and $\alpha = \frac{1}{2}$ [5]. Several series of experiments supported these predictions [1,2].

However, in more recent experiments, Koehler *et al.* rather find $\alpha \sim 0.36$ [4]. They suggest that the rigid wall condition may not be valid and they assume that the flow in the Plateau borders is pluglike. The dissipation is then dominated by the flow into the nodes that connect different Plateau borders. The solution of this hydrodynamic problem is again a soliton wave, with $V \sim Q^{\alpha}$ and $\alpha = \frac{1}{3}$, in excellent agreement with their experimental findings.

In the previous theoretical work, it was already noted that the rigid walls assumption could fail in some cases [5,6]. Indeed, aqueous foams are made with surfactant solutions, and they are stabilized by the surfactant monolayers adsorbed at the air-water interfaces. The rheological properties of these monolayers strongly depend upon surfactant nature and concentration. In order to check for their influence on forced drainage, different solutions of pure surfactants were studied and no significant variations of the flow velocity were observed [3]. Furthermore, it is not easy to distinguish experimentally between power law exponents $\frac{1}{2}$ and $\frac{1}{3}$. In Refs. [1–3], the range of flow rates was limited, less than two decades. The experimental procedure is also different from that of Koehler *et al.*: after forming the foam, gas bubbling is stopped, whereas Koehler *et al.* regenerate their foam continuously. In principle, the two procedures should be equivalent, but without gas bubbling, better foam stability is necessary to study different flow rates with the same foam. When reanalyzing the data of reference [3], we observed that when error bars are taken into account, the data are also consistent with $\alpha = \frac{1}{3}$. This prompted us to perform new measurements with a larger range of flow rates.

We have used a setup similar to that of Ref. [2]. The foam is formed by bubbling nitrogen through a porous disc in a plexiglas column with a squared section (4×4 cm) and height 70 cm. Series of electrodes (26 pairs) are equally spaced along the height of the column, in order to record the variation of electrical conductivity due to the passage of the liquid front. The typical bubble size is 1 mm [it cannot be controlled in our experiments where the foam is allowed to age and where bubble disproportionation (Oswald ripening) leads to a rapid growth of small bubbles]. We studied Dawn soap solutions identical to those used by Koehler *et al.* (concentration 0.25 wt %). For flow rates between 2×10^{-3} and 1.6 ml/s, we find $\alpha = 0.39 \pm 0.04$, in good agreement with their result (Fig. 1).

We then investigated pure surfactants solutions. In order to be able to span a large range of flow rates with the same foam, we used sodium dodecyl sulfate (SDS) solutions (12 mM, slightly above critical micellar concentration), and we incorporated minor amounts of dodecanol (weight ratio SDS/dodecanol, K, above 10^3). Dodecanol is indeed known to substantially increase the foam stability, by forming mixed monolayers with SDS at the air-water surface with high sur-

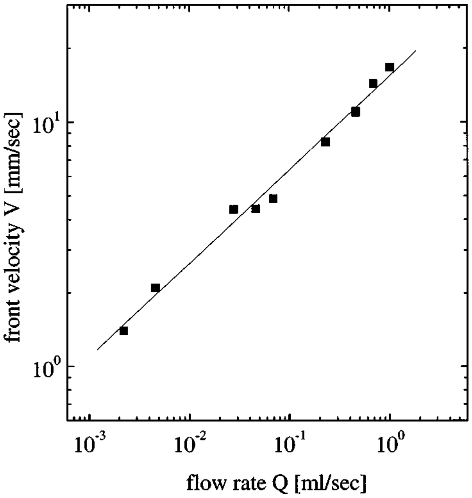

FIG. 1. Flow velocity vs flow rate in the forced drainage experiments with Dawn soap solutions.

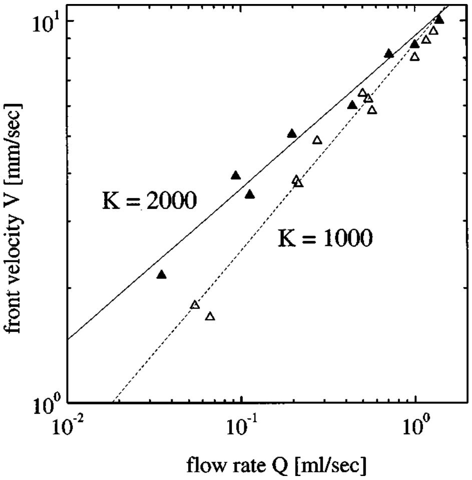

FIG. 2. Forced drainage results for mixed SDS-dodecanol solutions; K is the weight ratio SDS/dodecanol.

face viscosities [7,8]. The results are shown in Fig. 2. We find $\alpha = 0.54 \pm 0.03$ for $K = 10^3$, whereas $\alpha = 0.39 \pm 0.04$ for $K = 2 \times 10^3$.

When comparing the data for Dawn soap and for SDS/dodecanol, $K = 2 \times 10^3$, we see that the velocities are similar for a given shear rate. This is as expected: in a given regime when α is fixed, the velocity depends on the flow geometry, not on the particular value of the surface rheological parameters. This is probably why no significant variations of the flow velocity were observed in Ref. [3]. We do not know exactly which was the particular regime in these experiments; trial experiments with pure solutions of SDS or CTAB (cetyl trimethylammonium bromide, also used in [3]) showed that flow rates larger than those of Ref. [3] destabilize the foam. The range of flow rates can then not be extended sufficiently to determine precisely the value of α.

We thus confirm that there is an effect of the nature of the surfactant monolayer on the flow process. It is not yet clear whether the difference in behavior is due to differences between surface viscosities, and which surface viscosity needs to be considered, shear or dilational. The surface dilational elasticity E can also play a role via the surface Marangoni stresses. All three properties increase with dodecanol content in the monolayer [8]. From the hydrodynamic point of view, a single (isolated) surface behaves as a solid if $E/\gamma > 1$ or if $\kappa/\eta R > 10$, where γ is the surface tension, κ is the sum of the dilational and shear viscosities, η is the bulk viscosity and, R the bubble radius [9]. These results were extended to foam films for the influence of surface viscosities, and the results are similar [6]. When the film surface behaves as a solid, it is then natural to find a situation were the flow in Plateau borders is Poiseuille-like. On the contrary, when the film surface is fluid, it moves with the liquid and the flow should rather be pluglike. We are currently investigating foams where the three surface parameters, elasticity, shear, and surface dilational viscosity, are varied separately to clarify their role in the drainage process.

We are very grateful to Howard Stone and to his colleagues for constructive criticisms regarding the manuscript and the soap sample. We are also indebted to Denis Weaire and to Stefan Hutzler for numerous discussions and useful exchanges of information.

[1] D. Weaire, N. Pittet, S. Hutzler, and D. Pardal, Phys. Rev. Lett. **71**, 2670 (1993).

[2] S. Hutzler, G. Verbist, D. Weaire, and J. A. van der Steen, Europhys. Lett. **31**, 497 (1995).

[3] S. Stoyanov, C. Dushkin, D. Langevin, D. Weaire, and G. Verbist, Langmuir **14**, 4663 (1998).

[4] S. A. Koehler, S. Hilgenfeldt, and H. A. Stone, Phys. Rev. Lett. **82**, 4232 (1999).

[5] G. Verbist and D. Weaire, Europhys. Lett. **26**, 631 (1994); D. Weaire, S. Hutzler, G. Verbist, and E. A. J. Peters, Adv. Chem. Phys. **102**, 315 (1997).

[6] D. Desai and R. Kumar, Chem. Eng. Sci. **37**, 1361 (1982); A. M. Kraynik, Sandia Report No. SAND83-0844, 1983 (unpublished).

[7] A. M. Poskanzer and F. Goodrich, J. Phys. Chem. **79**, 2122 (1975).

[8] N. F. Djabbarah and D. T. Wasan, Chem. Eng. Sci. **37**, 175 (1982).

[9] V. G. Levich, *Physico-Chemical Hydrodynamics* (Prentice-Hall, Englewood Cliffs, NJ, 1962), p. 616; D. Langevin, *Light Scattering by Liquid Surfaces* (Marcel Dekker, New York, 1992), Chap. 11.